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I independently came up with an alternative way to obtain the sequence **A182541** and it allows me to suggest a possible interpretation for it. I can't prove in a formal way that the sequence suggested by me and **A182541**(n) are, in fact, the same, however, my formula correctly calculates all the listed terms for **A182541**(n) to consider it just a mere coincidence.

So let's consider the number of permutations of 1...n that have 2 following 1 for n >= 1 (A001710) as defined by Jon Perry 2008.

The sequence of matrices of the size (n+1) x (A182541(n+1)) can be constructed so that all the rows consist of permutations of 11...n

```
11,    112,                   1123                  11234     . . .
       121                    1132                  11243
       211                    1231                   ...
                              1213
                              ...
```

Then for each matrix we are going to do the following. For each element with the value of 'k' delete exactly k elements of this type from the matrix, repeat again if necessary until no elements with the value of k are left and the last element is reached. The whole operation shoud be repeated for all the values of k from 1 to n until no more elements can be deleted:

```
11    --> 1       1 element is left and there are no more ones to delete
=> n(1) = 1

112         102    102
121 -->     120 -> 100 4 non-zero elements are left => n(2)=4
211         210    010
```

**A182541(n) basically counts all the non-zero elements left in the matrix after the procedure of "deletion of elements" was completed as it was defined above.**

The deletion can be done in any order:

```
1123               1123                  1123             1023
1132               1102                  1100             1000
1231               1201                  1001             1000
1213               1210                  1210             1200
1321               1321                  1301             1300
1312 del 3 ->      1012   del 2-->       1010 del 1 ->    1000
2113               2110                  2110             2100
2131               2101                  0101             0100
3112               3112                  3110             3100
3121               0121                  0121             0120
3211               0211                  0011             0010
2311               2011                  0011             0010
```

19 elements are left. n(3) = 19

Clearly there is no need to draw all these matrices to get the A182541(n) as **the number of rows of any matrix of this type can be obtained directly from A001710 as the number of permutations.**

Finally, A182541(n) can be calculated by the means of formula suggested by me. These matrices have a special property, so that

[The Number of rows] =0 (mod[the number of k-elements left]) for every k =1 to n.

Initially each matrix has the same number of elements bigger than 1 and it equals the number of its rows.
It also has the number of elements with a value of 1 equal to the (number of rows)*2.

After the procedure of deletion of the elements is finished we got the number of 'ones' which is equal to the number of rows in the matrix or A182541(n+1). The number of twos left is equal to 1/3* (the number of rows) or 1/3 * A182541(n+1).
The number of threes left is equal to the 1/4*(number of rows) or 1/4 * A182541(n+1) and so on.

Basically, by definition,

a(1) = 1

a(2) = The number of non-zero elements left = Sum of all the numbers of all the non-zero elements left. = the number of all the ones left + the number of all the twos left = Number of rows + 1/3(Number of rows) = 3+3/3 = 3+1 = 4

a(3) = number of all the ones left + number of all the twos left + number of all the threes left = Number of rows + 1/3*(Number of rows) + 1/4*(Number of rows) = 12 +12/3 + 12/4 = 19 and so on.

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**a(1) =1**
**for n >=2 a(n)= A001710(n+1)*[1+Sum_{k=2..n}1/(k+1)]**

---

and we get the sequence:

1, 4, 19, 107, 702, 5274, 44712, 422568,...

The only noticeable difference between the two that can be observed is that A182541(n) defined by Sergey Kitaev, Jeffrey Remmel starts from a(3) = 1

and the 'related to A001710' sequence defined by me starts from a(1) = 1 what I believe may be even a bit more relevant, but anyway...

For coefficients in g.f. for certain marked mesh patterns as defined by Kitaev and Remmel the same formula will look like:

**a(3)=1 For n >= 4 a(n)=(A001710(n-1))*[1+ Sum_{k=2..n-2} 1/(k+1)]**

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Lastly, I want to mention that many well-known sequences can be generated with the help of the same operation defined above.

For instance, applying this rather odd and perhaps clumsy operation of 'deletion of the elements' to the type of (n+1)X(n+1) matrices based on the sequence.

112, 1113, 11114, 11115 ...

We get the Central Polygonal numbers. (OEIS A000124)

```
1113               1013
1131 -> ... -> 0100   = 7
1311               1001
3111               0010
```

7 non-zero elements are left and or the Central Polygonal number for n=3

```
11114     10104     10104      In the exact same way
11141     10140     10100      we may calculate a(4) = 11
11411 ->  10410 ->  10010      or the Central Polygonal number for n=4
14111     14010     10010
41111     4101      01010      and so on …
```

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If we consider the matrices of the type where the rows 1, 12, 123, 1234, 1234..n are repeated exactly n times and operation of deletion is applied several times we get **A123327**:

For instance,

```
1234       1234   Now we are going        1234
1234 ->    0004   to run the operation -> 0004
1234       1004   for the second time     0004    so a(4) = 8
1234       0204   to get rid of '1'       0204
```

```
12345     12345  The operation        12345  and for the    12345
12345 ->  00005  can be repeated      00005  last time ->   00005
1234      10005  again to get rid ->  00005  to get         00005   a(5) = 10
12345     02005  of 2 twos an         00005  rid of the     00005
12345     12305  1 one.               10305  last one       00305
```

Of course, it is rather trivial to be considered any seriously on its own and I added it just for an illustration purpose only to show that this operation may indeed have some sense/value.